17/11/2015
محاضرة 6 صفحة 9

Theories of elastic Failure

[1] Maximum Normal stress theory (Rankine).

→ The theory is based on Failure in tension or Compersion. This states that, Materail under combine stress reaches the limit of it's elastic Failure when, the maximum normal stress [Principle stress] reaches A value equal 2% yield stress of Material in simple tension or compression.

S_y = the largest of $\begin{cases} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{cases}$

For Brittle materail.

S_u = the Langest $\begin{cases} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{cases}$

Ex2- For Brittle materail the stress relation ship is non-linear and buinding stress are not exactly but the eqn. $\sigma = \frac{My}{I}$

Solution

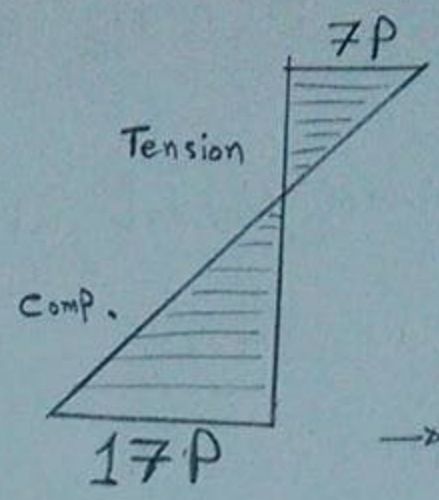
[b]

$$7P = \frac{S_{ut}}{F.S} = \frac{210}{2}$$

$$P = 15$$

$$AP = \frac{S_{uc}}{F.S.} = \frac{750}{2}$$

$$P = 22$$



C.I beam

$S_{ut} = 210$ MPa.

$S_{uc} = 750$ MPa.

F.S. = 2

→ P = ??

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[2] Max shear stress theory (Tresca).

→ These theory is used in designing ductile steal Machine Member. It's based on the assumbtion that the Failure accure when the maximum shear stress reaches a certain value that, value is the shear stress at the instant of Failure in tensile test.

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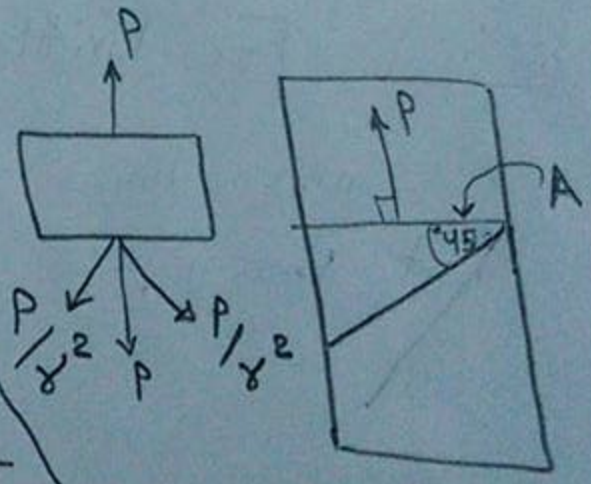
max. normal stress at yielding.

$$S_y = \frac{P}{A}$$

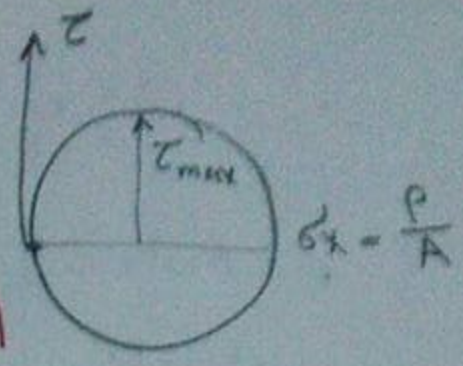
max. shear stress at yielding.

$$\tau_{max} = \frac{P/\sqrt{2}}{\sqrt{2} \cdot A} = \frac{P}{2A} = \frac{S_y}{2}$$



In case of (Triaxial stress.) :

$$\frac{Sy/2}{F.S} = \text{the largest of} \begin{cases} |(\sigma_1 - \sigma_2)| \\ |(\sigma_2 - \sigma_3)| \\ |(\sigma_3 - \sigma_1)| \end{cases}$$



$$\tau_{max} = \frac{\sigma_y}{2}$$

In case of (Biaxial stress) :

$$\frac{Sy/2}{F.S} = \text{the largest of} \begin{cases} \left|\frac{\sigma_1 - \sigma_2}{2}\right| \\ |\sigma_2/2| \\ |\sigma_1/2| \end{cases}$$

(1) → IF σ_1 and σ_2 are opposite sign ($\sigma_3 = 0$)

$$\frac{Sy/2}{F.S} = \frac{\sigma_1 - \sigma_2}{2}$$

(2) → IF σ_1 and σ_2 are of the same sign

$$\frac{Sy/2}{F.S} = \text{the largest} \begin{cases} \sigma_1 \\ \sigma_2 \end{cases}$$

d

3- Max. strain theory (st. Venont).

- Material under combine stress reachs the limit of elastic Failure when the maximum normal strain reaches value equal ℨ that yielding ie (يعني) yied strain tension or Compression.

$$\epsilon_y = \frac{\sigma_y}{E} = \frac{S_y}{E}$$

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - n\sigma_2)$$

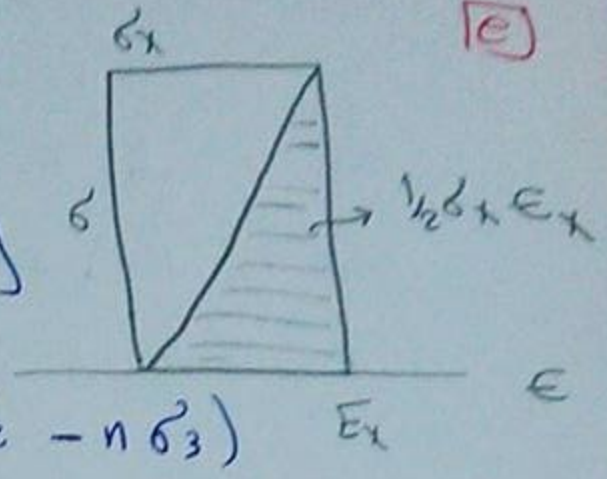
$$\epsilon_2 = \frac{1}{E}(\sigma_2 - n\sigma_1)$$

$$\frac{S_y}{F.S} = \text{the largest of } \begin{cases} |(\sigma_1 - n\sigma_2)| \\ |(\sigma_2 - n\sigma_1)| \end{cases}$$

4 - Max strain energy thory.

→ Material under combine stresses reaches the limit of elastic Failure when the energy per unit valume reaches avalue that equales 2 the energy per unit at yield stress at simple tension or Compression.

(e)



In case of triaxial.

$$U = \frac{1}{2}\left[\sigma_1 \epsilon_1 + \sigma_2 \epsilon_2 + \sigma_3 \epsilon_3\right]$$

total energy of $\epsilon_1 = \frac{1}{E}\left[\sigma_1 - n\sigma_2 - n\sigma_3\right)$

$$\epsilon_2 = \frac{1}{E}\left(\sigma_2 - n\sigma_1 - n\sigma_3\right)$$

$$\epsilon_3 = \frac{1}{E}\left(\sigma_3 - n\sigma_1 - n\sigma_2\right)$$

$$U = \frac{1}{2E}\left[\sigma_1 + \sigma_2 + \sigma_3 - 2n\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3\right)\right]$$

[5] Max. Distorsion Energy theory (Miss Hencky).

supject material to combine stress reaches the limit of elastic Failure when distorsion energy per unit volume reaches a value equals 2 that at yield stress is simple tension or compresion.

U_v = strain energy resulting From change in volume.

U_d = " " " From distorsion

U = total strain energy.

$$U = U_v + V_d \longrightarrow (1)$$

F

$$\sigma_1 = \sigma_{1d} + \sigma_V$$
$$\sigma_2 = \sigma_{2d} + \sigma_V$$
$$\sigma_3 = \sigma_{3d} + \sigma_V \quad \rightarrow (2)$$

where σ_{1d}, σ_{2d}, σ_{3d} are distorsims component

$$\epsilon_{1d} + \epsilon_{2d} + \epsilon_{3d} = 0 \quad \rightarrow (3)$$

$$\epsilon_{1d} = \frac{1}{E}\left[\sigma_{1d} - n(\sigma_{2d} + \sigma_{3d})\right]$$
$$\epsilon_{2d} = \frac{1}{E}\left[\sigma_{2d} - n(\sigma_{1d} + \sigma_{3d})\right]$$
$$\epsilon_{3d} = \frac{1}{E}\left[\sigma_{3d} - n(\sigma_1 d + \sigma_{2d})\right] \quad \rightarrow [4]$$

$$(1-2n)(\sigma_{1d} + \sigma_{2d} + \sigma_{3d}) = 0 \quad \rightarrow [5]$$

$$(\sigma_{1d} + \sigma_{2d} + \sigma_{3d}) = 0 \quad \rightarrow [6]$$

From ② & ⑥

$$\sigma_V = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \rightarrow [7]$$

$$\epsilon_V = \frac{1}{E}(\sigma_V - n(\sigma_V + \sigma_V) = \frac{1-2n}{E}\sigma_V \quad \rightarrow [8]$$

$$U_V = \frac{3}{2}\frac{(1-2n)}{E}\sigma_V^2 \quad \rightarrow [9]$$

$$U_V = 3\frac{(1-2n)}{E} \cdot \sigma_V \cdot \frac{\sigma_V}{2}$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2n\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3\right)\right]$$

$$U_V = \frac{(1-2n)}{6E}\left(\sigma_1 + \sigma_2 + \sigma_3\right)^2$$

$$U_V = \frac{(1-2n)}{6E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_1\sigma_3\right]$$

↳ (10)

$$U_d = U - U_V$$

$$U_d = \frac{(1+m)}{6E}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

$$\sigma = S_y$$

$$\boxed{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1) = 2S_y^2}$$

For Biaxial

$$\boxed{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_y}$$

(h) Notes on the theories of Failue

→ let $x = \frac{\sigma_1}{sy}$ $y = \frac{\sigma_y}{sy}$

For max Normal

$\sigma_1 = \pm sy$ $x = \pm 1$

$\sigma_2 = \pm sy$ $y = \pm 1$

For max shear

IF σ_1, σ_2 of sam sign

$\sigma_1 = \pm \sigma y$ $x = \pm 1$

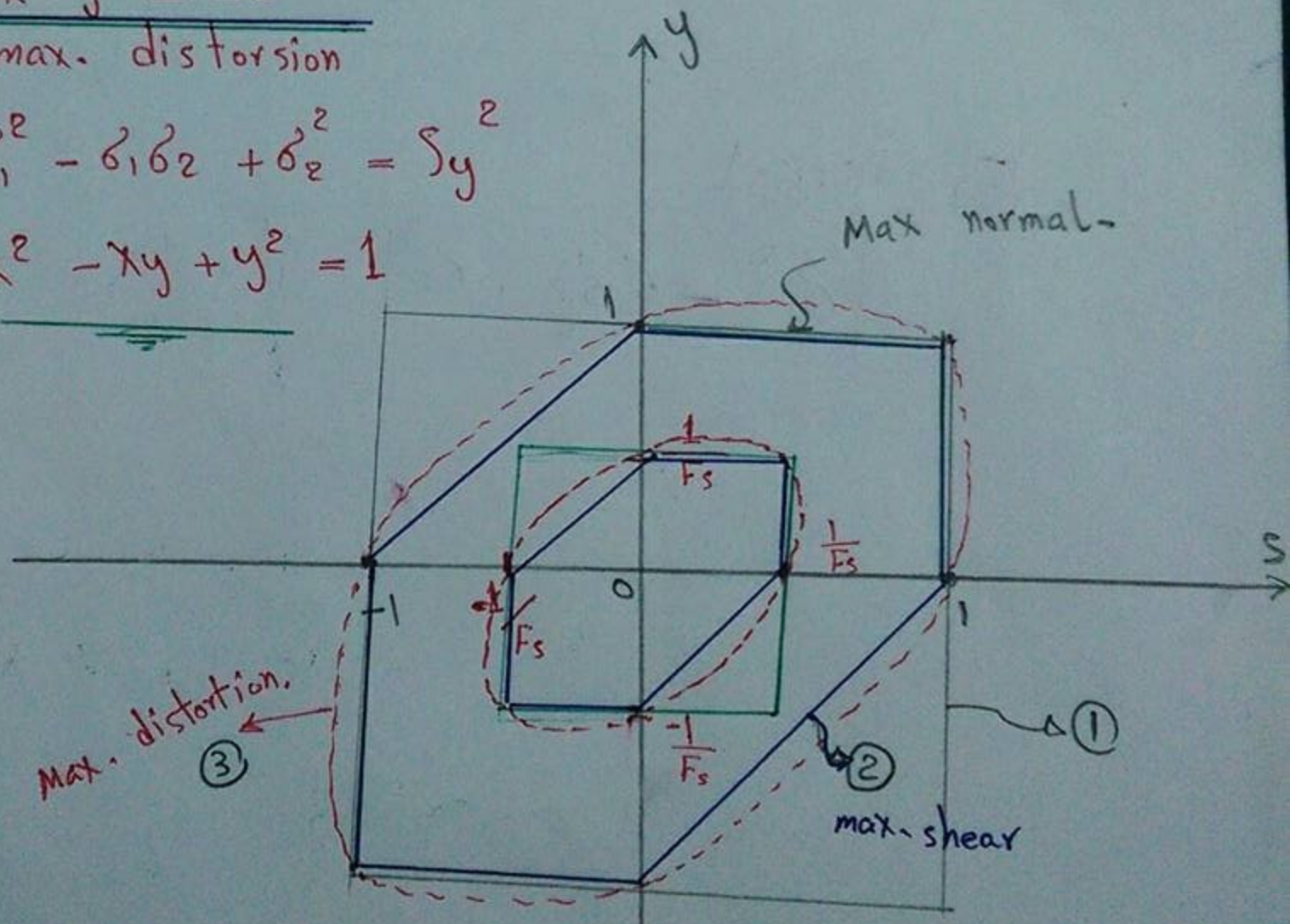
$\sigma_2 = \pm sy$ $y = \pm 1$

if σ_1 & σ_2 of the opposite

$(\sigma_1 - \sigma_2) = \pm sy$

$x - y = \pm 1$

For max. distorsion

$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = sy^2$

$x^2 - xy + y^2 = 1$